# Probing for variation of neutrino mass with current observations

Gong-Bo Zhao, Jun-Qing Xia, and Xinmin Zhang
*Institute of High Energy Physics, Chinese Academy of Science, P.O. Box 918-4, Beijing 100049, P. R. China*

With the latest astronomical data including Cosmic Microwave Background (WMAP three year, CBI, ACBAR, VSA), Type Ia Supernova ("gold sample"), galaxy clustering (SDSS 3-D matter power, Lyman-$\alpha$ forest and Baryon Acoustic Oscillating (BAO)), we make a global fitting to constrain the mass varying neutrinos. We find the parameter denoting time evolving of neutrino mass $\delta$ is weakly constrained and the neutrino mass limit today can be relaxed at least by a factor of 2. Adding data of $0\nu2\beta$ decay of Heidelberg-Moscow experiment to our analysis, we find $\delta$ can be measured and mass varying neutrinos are favored at about 99.7% confidence level.

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The mass varying neutrino which has been studied actively in the literature recently brings forth new theoretical challenges and abundant phenomenology. The predictions of the variation of the neutrino masses can be tested in the experiments of neutrino oscillation[1], short gamma ray bursts[2] and extremely high energy cosmic neutrino[3]. Cosmologically the mass varying neutrino plays an interesting role in determining the evolution of the universe[4] and has interesting implications in leptogenesis[5], Supernova[6], the Cosmic Microwave Background(CMB) and Large Scale Structure(LSS)[7].

Neutrino mass variation can be induced by interaction between neutrino and scalar field[8, 9, 10, 11] in models of neutrino dark energy. In the framework of ΛCDM the variation of neutrino mass could be resulted from coupling between neutrino and Ricci scalar in form of $f(R)\bar{\nu}\nu$ and possibly some other unveiled mechanism. In this *Letter* we work in a model independent way to constrain the time evolving of neutrino mass by parameterizing neutrino mass in the form of Eq.(1). For simplicity and to purify the effect of mass varying neutrinos we use the simplest dark energy model, the cosmological constant for the analysis in this paper.

We take a global fit to constrain parameter, $\delta$ denoting the mass variation of neutrinos and study its effect on the determination of other cosmological parameters. Our result shows that 1)with the mass variation, the cosmological limit on the neutrino mass is relaxed by roughly a factor of 2; 2)if taking the limits on the neutrino mass from Heidelberg-Moscow experiment(HM) as a prior[12, 13, 14], we find the variation of neutrino mass is tightly constrained and the mass varying neutrinos are favored at about 3 $\sigma$.

We assume three species of neutrinos whose masses are degenerate and parameterize the neutrino mass in the form of

$$m_\nu(a) = m_{\nu0}[1 + \delta(1 - a)] \tag{1}$$

where $m_\nu(a)$ is the sum of neutrino masses which is a function of scale factor $a$, $m_{\nu0}$ is the sum of neutrino mass at present, $\delta$ is the dimensionless factor denoting the time-varying effect of neutrino masses. For the background,

$$\rho_\nu = \frac{1}{a^4}\int q^2 dq d\Omega \epsilon f_0(q), \tag{2}$$

with $\epsilon^2 = q^2 + m_\nu(a)^2 a^2$, $q^i = ap^i$ is the comoving momentum. The pressure is

$$p_\nu = \frac{1}{3a^4}\int q^2 dq d\Omega f_0(q)\frac{q^2}{\epsilon}. \tag{3}$$

Thus,

$$\dot{\rho}_\nu + 3H(\rho_\nu + p_\nu) = \frac{\partial \ln m_\nu(a)}{\partial a}\dot{a}(\rho_\nu - 3p_\nu). \tag{4}$$

where the overdot represents the derivative with respect to conformal time. For the evolution of perturbation, working in the synchronous gauge and following the treatment of Ref.[15], we find the Boltzman equations Eq.(40) in [15] does not relate directly to mass varying terms $\partial \ln m_\nu/\partial a$ therefore the hierarchy equations for massive neutrinos Eq.(56) in [15] remains unchanged. In our numerical calculation we integrate these hierarchy equations rather than use the approximate strategy suggested in [16] to assure the high precision. Using a modified code of `CosmoMC`[17][1], we make a global fit to constrain the mass varying neutrinos. Assuming a flat universe, we set the most general cosmological parameter space as:

$$\mathbf{p} \equiv (\omega_b, \omega_c, \Theta_S, \tau, f_\nu, \delta, n_s, \log[10^{10}A_s]) \tag{5}$$

where $\omega_b = \Omega_b h^2$ and $\omega_c = \Omega_c h^2$ are the physical baryon and cold dark matter densities relative to critical density, $\Theta_S$ characterizes the angular scale of sound horizon, $\tau$ is the optical depth to the last scattering surface and $A_s$ is defined as the amplitude of initial power spectrum, $f_\nu$ is dark matter neutrino fraction at present, namely,

$$f_\nu = \rho_\nu/\rho_{DM} = \frac{m_{\nu0}}{93.105\ eV\ \Omega_c h^2}\ , \tag{6}$$

[1] Available at http://cosmologist.info/cosmomc/

where $\rho_\nu$ and $\rho_{DM}$ denote the energy density of neutrino at present and dark matter respectively. We sample in above 8 dimensional parameter space and fit the theoretical output to the observation using Markov Chain Monte Carlo algorithm[22, 23, 24]. We take the weak priors as: $\tau < 0.8$, $0.5 < n_s < 1.5$, $0 < f_\nu < 0.5$, a cosmic age tophat prior as 10 Gyr< $t_0$ <20 Gyr. To keep the positivity of neutrino mass we take -1< $\delta$ <10. Furthermore, we make use of the HST measurement of the Hubble parameter $H_0 = 100h$ km s$^{-1}$Mpc$^{-1}$ [29] by multiplying the likelihood by a Gaussian likelihood function centered around $h = 0.72$ and with a standard deviation $\sigma = 0.08$. We impose a weak Gaussian prior on the baryon density $\Omega_b h^2 = 0.022 \pm 0.002$ ($1\sigma$) from Big Bang nucleosynthesis[30].

For CMB data, we use the three year WMAP (WMAP-3) Temperature-Temperature (TT) and Temperature-Polarization (TE) power spectrum with the routine for computing the likelihood supplied by the WMAP team [18, 19, 20, 21][2] as well as ACBAR [25], CBI [26, 27] and VSA [28] data. To break the degeneracy of cosmological parameters, we add non-CMB data into our analysis. For supernova type Ia (SN Ia) of "Riess gold sample" [31], we have marginalized over the nuisance parameter[33] in the calculation of SN Ia likelihood. For LSS information, we have used the 3-D matter power spectrum of SDSS[35] and 2dFGRS [34], Lyman-$\alpha$ forest data (Ly$\alpha$) from SDSS [37] and recent measurement of the baryon acoustic oscillation feature in the 2-point correlation function of SDSS [38]. To be conservative but more robust, we only use the first 14 bins of the SDSS 3-D matter power spectrum, which are well within the linear regime[39]. For Ly$\alpha$ likelihood, we modify the interpolating code[3] to incorporate our models. For BAO likelihood, we use the constraint [38]:

$$A \equiv D_V(0.35)\frac{\sqrt{\Omega_m H_0^2}}{0.35c} = 0.469 \pm 0.017 \quad , \qquad (7)$$

$$D_V(z) = \left[D_M(z)^2 \frac{cz}{H(z)}\right]^{1/3} \quad , \qquad (8)$$

where $H(z)$ is the Hubble parameter, c is the speed of light and $D_M(z)$ is the comoving angular diameter distance at a specific redshift $z$. Moreover, the Heidelberg-Moscow experiment uses the half time of $0\nu 2\beta$ decay to constrain the effective Majorana mass and this translates to the constraints of sum of neutrino mass under some assumptions[40]:

$$m_{\nu 0} \sim 1.8 \pm 0.6 \quad eV \quad (2\sigma). \qquad (9)$$

TABLE I: Mean and 1-$\sigma$ constraints of cosmological parameters. For the weakly constrained parameters, such as $m_{\nu 0}$ and $\delta$ for some data combination, we quote the 95% upper limits instead. Upper part of the table is for $\Lambda CDM$ + neutrinos with constant mass while in the lower part we free $\delta$ to study mass varying neutrinos. "ALL" denotes WMAP3+ACBAR+VSA+CBI+RIESS+SDSS+2dF+Ly$\alpha$ throughout this paper.

| $\delta = 0$ | ALL | ALL+BAO | ALL+BAO+HM |
|---|---|---|---|
| $m_{\nu 0}$ (eV) | $< 0.616$ | $< 0.393$ | $0.760^{+0.093}_{-0.104}$ |
| $\Omega_m$ | $0.317 \pm 0.021$ | $0.280 \pm 0.015$ | $0.303^{+0.016}_{-0.017}$ |
| $\sigma_8$ | $0.832 \pm 0.024$ | $0.834 \pm 0.024$ | $0.795^{+0.025}_{-0.026}$ |
| $\delta$ free | ALL | ALL+BAO | ALL+BAO+HM |
| $\delta$ | $< 6.661$ | $< 8.562$ | $< -0.713$ |
| $m_{\nu 0}$ (eV) | $< 1.619$ | $< 0.776$ | $1.568^{+0.143}_{-0.141}$ |
| $\Omega_m$ | $0.319 \pm 0.024$ | $0.281 \pm 0.014$ | $0.298 \pm 0.015$ |
| $\sigma_8$ | $0.829 \pm 0.027$ | $0.835^{+0.025}_{-0.024}$ | $0.790^{+0.022}_{-0.023}$ |
| $\chi^2$ reduced | 0.198 | 0.222 | 9.752 |

Given the Heidelberg-Moscow experiment is controversial for the time being, we just make a tentative fit choosing the HM prior.

For each regular calculation, we run 6 independent chains comprising of 150,000-300,000 chain elements and spend thousands of CPU hours to calculate on a cluster. The average acceptance rate is about 40%. We eliminate the first 10% chains elements for "build in", and for the convergence test typically we get the chains satisfy the Gelman and Rubin[41] criteria where R-1<0.1.

We summarized our main results of mass varying neutrinos in lower part of Table.I. For comparison, we also study the $\Lambda$CDM model with neutrinos of constant mass. The first discovery is that the neutrino mass limit at present epoch can be relaxed dramatically if neutrino mass varies during evolution. Without Heidelberg-Moscow data, we find the neutrino mass limit can be relaxed by a factor of 2.6 for "All" data and 1.8 for "All+BAO". Adding Heidelberg-Moscow prior, we find the mean value of neutrino mass rises up from 0.760 to 1.568 while reducing $\chi^2$ by 9.752. This is expected since the Heidelberg-Moscow prior is in great tension with cosmological observations. For example, the authors of Ref.[42] argued given current cosmological constraint on the (constant) neutrino mass the HM prior can be excluded. However, this controversy can be resolved if neutrino mass varies, for our *ad hoc* parametrization (1), we see All+BAO+HM prior can put stringent limits on $\delta$, namely, $-1 < \delta < -0.713$. The best fit value of $\delta$ is about -0.9. That means neutrino mass is very small in the past to be consistent with all the cosmological observation and grows heavy recently to fit the Heidelberg-Moscow data.

Mass varying neutrinos lessen the tension between HM prior and cosmological data thus are favored at nearly 3-

[2] Available at http://lambda.gsfc.nasa.gov/product/map/current/
[3] Available at http://www.cita.utoronto.ca~pmcdonal/LyaF/public.lyaf.tar.gz

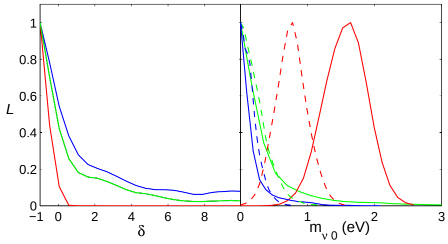


FIG. 1: One dimensional posterior distribution of neutrino mass today $m_0$ and its time variation $\delta$. Solid curves denotes the mass varying models (see text) while dashed lines show the models with constant neutrino mass. Different data combinations are distinguished by color. Green: All; Blue: All+BAO; Red: All+BAO+HM prior.

$\sigma$ than neutrinos with constant mass. These results are shown graphically in Fig.1. From the left panel we see $\delta$ is weakly constrained unless we add HM prior while right panel shows explicitly the modification of posterior distribution of neutrino mass today if we allow it to vary with cosmic time. Further, from Fig.2 we see current neutrino mass is correlated with its time variation. This correlation mainly stems from LSS data. We know galaxy survey are powerful to weigh neutrinos by detecting the suppression on small scales due to the free streaming effect of neutrinos[43]. The free streaming scale of mass varying neutrinos $\lambda_{FS-\nu}$ can be roughly estimated as:

$$\lambda_{FS-\nu} \simeq 20(\frac{m_\nu(a_{NR})}{30eV})^{-1}\ Mpc \qquad (10)$$

where $a_{NR}$ is the scale factor when neutrino becomes non-relativistic. We have seen from Eq.(10) and (1), $\lambda_{FS-\nu}$ is determined by the neutrino mass today and its evolution behavior, which leads to the correlation among $m_{\nu 0}$, $\delta$ and $m_{\nu 0}*\delta$. In Fig.2 we find $m_{\nu 0}$ is anti-correlated with $\delta$ and $m_{\nu 0}*\delta$ as expected.

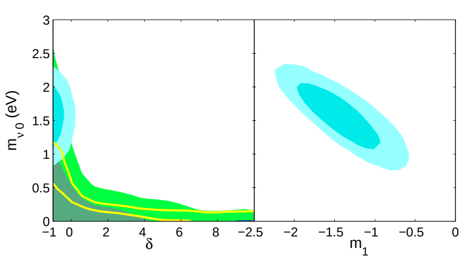


FIG. 2: Contour plots of parameters related to mass varying effect of neutrino. $m_1$ is neutrino mass changing with time, say, $m_1 = m_0 * \delta$. Different data combinations are distinguished by color. Green: All; Yellow: All+BAO; Cyan: All+BAO+HM prior. 68% and 95% C.L. contours are illustrated from inside out.

From Table.I, we find the mean value and error bars of $\Omega_m$ and $\sigma_8$ do not change much if neutrino mass varies. At first glance this seems at odds since the neutrino mass limit today has been significantly relaxed by its time variation and we know the neutrino mass today is strongly correlated with matter density and $\sigma_8$ as illustrated in Fig.(3). However since $\delta$ is anti-correlated with $m_{\nu 0}$, the aforementioned effect is counteracted, leaves $\Omega_m$ and $\sigma_8$ nearly unchanged. This means mass varying neutrinos can hardly be excluded by data sensitive to $\Omega_m$ and $\sigma_8$, such as SN Ia, CMB and LSS.

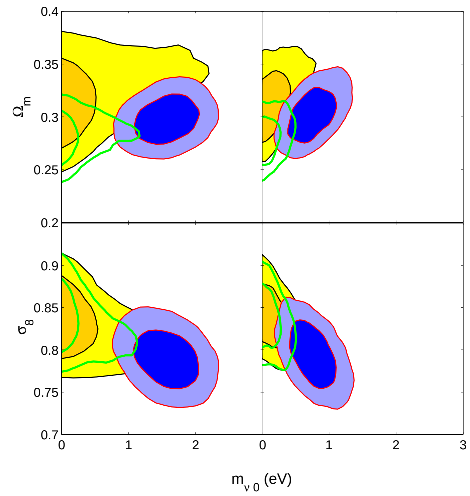


FIG. 3: Contour plots of sum of neutrino mass at present versus $\Omega_m$ and $\sigma_8$. Left panel: Neutrino mass varies as Eq.1; Right panel: Constant neutrino mass. Different data combinations are distinguished by color. Yellow: All; Green: All+BAO; Blue: All+BAO+HM prior. 68% and 95% C.L. contours are illustrated from inside out.

In Fig.(4), we show the evolution of neutrino mass with 1 and 2 $\sigma$ error using all data mentioned in this *Letter*. we see neutrino mass is best measured at a intermediate redshift rather than now due to the sensitive SN data in this redshift range.

In summary we in this *Letter* for the first time study the cosmological implications of time variation of neutrino mass in a model-independent fashion. We find numerically time variation of neutrino mass can relax the current mass limit significantly. This result has some interesting and important implications. For instance, it could thus resolve the tension between HM prior and cosmological data while does not spoil the common prediction of $\Omega_m$ and $\sigma_8$, and it might also be possible to revive the models of warm dark matter which has been shown to be excluded[44].

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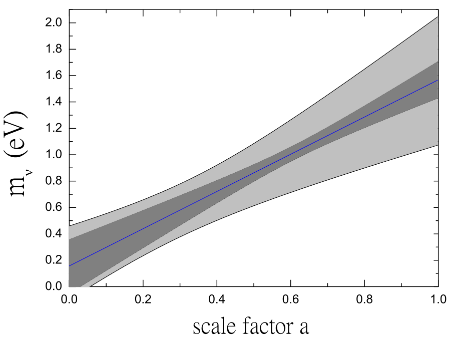


FIG. 4: Evolution of neutrino mass with respect to scale factor. From inside out, we show the models with mean parameter value (central blue line), 68% C.L.(dark gray) and 95% C.L. (bright gray) fitting with the data combination of All+BAO+HM prior.

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[1] D. B. Kaplan, A. E. Nelson, N. Weiner, Phys. Rev. Lett. **93**, 091801(2004); V. Barger, P. Huber and D. Marfatia, hep-ph/0502196; M. Cirelli, M. C. Gonzalez-Garcia and C. Pena-Garay, Nucl. Phys. B**719**, 219(2005); V. Barger, D. Marfatia, K. Whisnant, hep-ph/0509163; P. H. Gu, X. J. Bi, B. Feng, B. L. Young and X. Zhang, arXiv:hep-ph/0512076.

[2] H. Li, Z. g. Dai and X. Zhang, Phys. Rev. D **71**, 113003 (2005).

[3] A. Ringwald and L. Schrempp, JCAP **0610**, 012 (2006).

[4] X. J. Bi, B. Feng, H. Li and X. Zhang, Phys. Rev. D **72**, 123523 (2005).

[5] X. J. Bi, P. h. Gu, X. l. Wang and X. Zhang, Phys. Rev. D **69**, 113007 (2004); P. h. Gu and X. j. Bi, Phys. Rev. D **70**, 063511 (2004).

[6] H. Li, B. Feng, J. Q. Xia and X. Zhang, Phys. Rev. D **73**, 103503 (2006).

[7] A. W. Brookfield, C. van de Bruck, D. F. Mota and D. Tocchini-Valentini, Phys. Rev. Lett. **96**, 061301 (2006); A. W. Brookfield, C. van de Bruck, D. F. Mota and D. Tocchini-Valentini, Phys. Rev. D **73**, 083515 (2006).

[8] P. Gu, X. Wang and X. Zhang, Phys. Rev. D **68**, 087301 (2003).

[9] R. Fardon, A. E. Nelson and N. Weiner, JCAP **0410**, 005 (2004).

[10] R. D. Peccei, Phys. Rev. D **71**, 023527 (2005).

[11] For a brief review, see X. Zhang, AIP Conf. Proc. **805**, 3 (2006).

[12] H.V. Klapdor-Kleingrothaus, I.V. Krivosheina, A. Dietz, and O. Chkvorets, Phys. Lett. B **586**, 198 (2004).

[13] H.V. Klapdor-Kleingrothaus, talk at *SNOW 2006*, 2nd Scandinavian Neutrino Workshop (Stockholm, Sweden, 2006).

[14] G. L. Fogli *et al.*, hep-ph/0608060.

[15] C. P. Ma and E. Bertschinger, Astrophys. J. **455**, 7 (1995).

[16] A. Lewis and A. Challinor, Phys. Rev. D **66**, 023531 (2002).

[17] A. Lewis and S. Bridle, Phys. Rev. D **66**, 103511 (2002).

[18] D. N. Spergel *et al.*, arXiv:astro-ph/0603449.

[19] L. Page *et al.*, arXiv:astro-ph/0603450.

[20] G. Hinshaw *et al.*, arXiv:astro-ph/0603451.

[21] N. Jarosik *et al.*, arXiv:astro-ph/0603452.

[22] D. Gamerman, *Markov Chain Monte Carlo: Stochastic simulation for Bayesian inference* (Chapman and Hall, 1997).

[23] D. J. C. MacKay (2002), `http://www.inference.phy.cam.ac.uk/mackay/itprnn/book.html`.

[24] R. M. Neil (1993), `ftp://ftp.cs.utoronto.ca/pub/~radford/revie`

[25] C. l. Kuo *et al.* [ACBAR collaboration], Astrophys. J. **600**, 32 (2004)

[26] T. J. Pearson et al. (2002), astro-ph/0205388.

[27] CBI Supplementary Data, 10 July (2002), `http://www.astro.caltech.edu/~tjp/CBI/data/`.

[28] P. F. Scott et al. (2002), astro-ph/0205380.

[29] W. L. Freedman *et al.*, Astrophys. J. **553**, 47 (2001).

[30] S. Burles, K. M. Nollett and M. S. Turner, Astrophys. J. **552**, **L**1 (2001).

[31] A. G. Riess *et al.* [Supernova Search Team Collaboration], Astrophys. J. **607**, 665 (2004).

[32] A. Clocchiatti *et al.* (the High Z SN Search Collaboration), astro-ph/0510155.

[33] For details see e.g. E. Di Pietro and J. F. Claeskens, Mon. Not. Roy. Astron. Soc. **341**, 1299 (2003).

[34] S. Cole *et al.* [The 2dFGRS Collaboration], Mon. Not. Roy. Astron. Soc. **362** (2005) 505.

[35] M. Tegmark *et al.* [SDSS Collaboration], Astrophys. J. **606**, 702 (2004).

[36] M. Tegmark *et al.* [SDSS Collaboration], Phys. Rev. D **69**, 103501 (2004).

[37] P. McDonald *et al.*, arXiv:astro-ph/0407377.

[38] D. J. Eisenstein *et al.*, Astrophys. J. **633**, 560 (2005).

[39] M. Tegmark *et al.* [SDSS Collaboration], Phys. Rev. D **69**, 103501 (2004).

[40] A. De La Macorra, A. Melchiorri, P. Serra and R. Bean, arXiv:astro-ph/0608351.

[41] A. Gelman and D. Rubin, Statistical Science **7**, 457 (1992).

[42] U. Seljak, A. Slosar and P. McDonald, arXiv:astro-ph/0604335.

[43] W. Hu, D. J. Eisenstein and M. Tegmark, Phys. Rev. Lett. **80**, 5255 (1998).

[44] U. Seljak, A. Makarov, P. McDonald and H. Trac, arXiv:astro-ph/0602430.